

# Phase textures induced by dc current pairbreaking in multilayer structures and two-gap superconductors.

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We predict current-induced formation of equilibrium phase textures for a multicomponent superconducting order parameter. Using the two-component Ginzburg-Landau and Usadel equations, we show that, for weakly coupled co-moving superconducting condensates, dc current  $I$  first causes breakdown of the phase locked state at  $I > I_{c1}$  followed by the formation of intrinsic phase textures well below the depairing current  $I_d$ . These phase textures can manifest themselves in multilayer structures, atomic Bose condensate mixtures in optical lattices and two-gap superconductors, particularly  $\text{MgB}_2$ , where they can result in oscillating and resistive switching effects.

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Interest in novel effects caused by multicomponent order parameters in heavy fermion and organic superconductors [1] has been recently amplified by the discovery of two-gap superconductivity in  $\text{MgB}_2$  [2], which exhibits an anomalous increase of the upper critical field by impurities [3] and intrinsic Josephson effect between two weakly coupled order parameters  $\Psi_1 = \Delta_1 e^{i\theta_1}$  and  $\Psi_2 = \Delta_2 e^{i\theta_2}$  in  $\sigma$  and  $\pi$  bands. Excitations of the interband phase difference  $\theta = \theta_1 - \theta_2$  can be either the phonon-like Leggett modes [4] or phase textures formed by  $2\pi$  interband phase slips. Phase textures and peculiar vortex properties [5] in two-gap superconductors are generic manifestations of the multicomponent nature of the order parameter, which have analogs in superfluid  $^3\text{He}$  [4] and models of color superconductivity in the particle physics [6].

Spontaneous formation of phase textures breaking the time reversal symmetry is inhibited by interband coupling, which locks the phases of  $\Psi_1 = \Delta_1 e^{i\theta_1}$  and  $\Psi_2 = \Delta_2 e^{i\theta_2}$ . However, for weak interband coupling characteristic of  $\text{MgB}_2$ , the order parameters  $\Psi_1$  and  $\Psi_2$  can be unlocked by electric fields. The resulting *non-equilibrium* charge imbalance generates interband phase textures, which do not carry magnetic flux and thus do not interact with weak magnetic fields and supercurrents [7]. In this Letter we show that, for sufficiently strong superconducting currents, formation of phase textures does not necessarily require any nonequilibrium conditions and can result from dc current pairbreaking, which decouples  $\Psi_1$  and  $\Psi_2$  well below the global depairing threshold. Thus, the phase textures are indeed a rather generic feature of *equilibrium* current-carrying states, which are not specific to two-gap superconductors, but can be realized in any system with at least two different co-moving superconducting or Bose condensates, in either the coordinate or the momentum space. The examples range from a weakly coupled thin film bilayer (Fig. 1) to a mixture of two weakly coupled atomic Bose condensates in optical lattices [8]. The bilayer in Fig. 1 can be mapped onto a two-gap superconductor in which two electron bands cor-

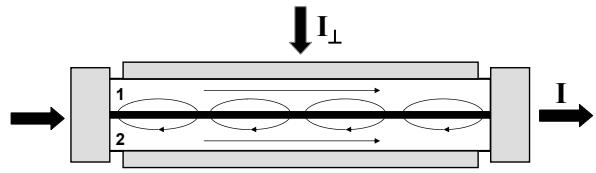


FIG. 1: Formation of phase textures in current-carrying bilayer. Thick black line shows the interlayer Josephson contact and gray rectangles show current leads.

respond to the different films, and the interband coupling corresponds to the interlayer Josephson energy.

The mechanism of texture formation is particularly transparent in a bilayer, which also reveals peculiar non-stationary effects such as switching, parametric amplification, and flux flow oscillations controlled by the applied current in the absence of a dc magnetic field. The sheet supercurrent  $I = I_1 + I_2$  in the bilayer is sustained by the phase gradients  $\theta'_1$  and  $\theta'$  in films 1 and 2, where  $I_1 = -cd_1\phi_0\theta'_2/8\pi^2\Lambda_1^2$ ,  $I_2 = -cd_2\phi_0\theta'_1/8\pi^2\Lambda_2^2$ ,  $\Lambda_1$  and  $\Lambda_2$  are the London penetration depths,  $\phi_0$  is the flux quantum,  $c$  is the speed of light, and the vector potential  $\mathbf{A}$  is negligible for small film thicknesses  $d_1 \ll \Lambda_1$  and  $d_2 \ll \Lambda_2$ . For small current  $I$ , the minimum energy corresponds to the phase-locked state with the global phase gradient  $\theta'_1 = \theta'_2 = Q = -8\pi^2(\Lambda_1^2/d_1 + \Lambda_2^2/d_2)I/c\phi_0$  for which the interlayer Josephson energy vanishes.

The situation changes at higher  $Q \sim 1/\xi_2$ , if the coherence length  $\xi_2$  in film 2 is greater than  $\xi_1$  in film 1. Then the Ginzburg-Landau (GL) current pairbreaking suppresses the gap  $\Delta_2(I) = \Delta_2[1 - (\xi_2 Q)^2]^{1/2}$  more than the other gap  $\Delta_1(I) = \Delta_1[1 - (\xi_1 Q)^2]^{1/2}$ , so the maximum phase-locked current is limited by the depairing threshold  $Q = 1/\sqrt{3}\xi_2$  in film 2. For  $Q > 1/\sqrt{3}\xi_2$ , film 2 goes normal, forcing current redistribution, which restores superconductivity in film 2, but causes a gradient of the phase difference  $\theta'(x)$  and the oscillating perpendicular Josephson current  $J_c \sin \theta$  along the bilayer. As a

result, current loops appear in which currents flow parallel to  $I$  in the "stronger" film 1 and antiparallel to  $I$  in the "weaker" film 2, as shown in Fig. 1. Such decoupling transition occurs if the gain in the condensation energy due to the current redistribution exceeds the loss in the Josephson energy for a single vortex.

A theory of this transition can be developed using the GL free energy  $\mathfrak{F} = \int F d^2\mathbf{r}$ ,

$$F = \sum_m d_m \left[ -\tilde{\alpha}_m \Delta_m^2 + \frac{\beta_m}{2} \Delta_m^4 + \gamma_m (\nabla \Delta_m)^2 \right] + B^2 / 8\pi + \epsilon_J (1 - \cos \theta), \quad (1)$$

where  $m$  labels films 1 and 2,  $\tilde{\alpha}_m = \alpha_m - Q_m^2 \gamma_m$ ,  $\mathbf{Q}_m = \nabla \theta_m + 2\pi \mathbf{A}/\phi_0$ , and  $\mathbf{B} = \nabla \times \mathbf{A}$ . The last term defines the sheet coupling energy  $\epsilon_J = \phi_0 J_c / 2\pi c$  where  $J_c$  is the Josephson critical current density. For weak coupling,  $\theta(x)$  varies over the length  $L_\theta$  much greater than  $\xi_m = (\gamma_m/\alpha_m)^{1/2}$ . Thus,  $\Delta_m^2 = \tilde{\alpha}_m/\beta_m$  to the zero accuracy in  $\epsilon_J$ , and the thermodynamic potential  $G$  for slowly varying  $Q_m(x)$  and fixed  $I$ , takes the form:

$$G = \int d^2\mathbf{r} \left[ -\sum_m \frac{d_m \alpha_m^2}{2\beta_m} (1 - Q_m^2 \xi_m^2)^2 + \epsilon_J (1 - \cos \theta) + IA/c - \phi_0 J_\perp \theta / 2\pi c \right]. \quad (2)$$

Here  $J_\perp$  is the current density injected *perpendicular* to the layers, and the sheet current  $I(Q)$  along the bilayer is determined from  $\partial G / \partial A = 0$ . Next, we extract the  $\theta$ -dependent energy  $G_\theta$  of phase textures, expressing  $Q_m$  in terms of  $\theta$  and  $I$  in Eq. (2). For slow variations of  $\theta(x)$ , a quadratic gradient expansion of  $G_\theta$  yields

$$G_\theta = \epsilon_J \int \left[ \frac{L_\theta^2}{2} (\nabla \theta)^2 + 1 - \cos \theta - \eta \theta - (\mathbf{Q} \nabla \theta) h \right] d^2\mathbf{r}, \quad (3)$$

where  $\eta = J_\perp/J_c$ . The phase length  $L_\theta(Q)$ , the coupling parameter  $h(Q)$  and  $I(Q)$  depend parametrically on the background gauge-invariant phase gradient  $\mathbf{Q}$ :

$$h = \frac{8\epsilon_1\epsilon_2(\xi_2^2 - \xi_1^2)Q^2}{[(1 - 3\xi_1^2Q^2)\epsilon_1 + (1 - 3\xi_2^2Q^2)\epsilon_2]\epsilon_J} \quad (4)$$

$$L_\theta^2 = \frac{4\epsilon_1\epsilon_2(1 - 3\xi_1^2Q^2)(1 - 3\xi_2^2Q^2)}{[(1 - 3\xi_1^2Q^2)\epsilon_1 + (1 - 3\xi_2^2Q^2)\epsilon_2]\epsilon_J} \quad (5)$$

$$I = -Q[\epsilon_1(1 - \xi_1^2Q^2) + \epsilon_2(1 - \xi_2^2Q^2)]8\pi c/\phi_0. \quad (6)$$

Here  $\epsilon_1 = d_1 \xi_1^2 \alpha_1^2 / 2\beta_1$  and  $\epsilon_2 = d_2 \xi_2^2 \alpha_2^2 / 2\beta_2$  are characteristic condensation energies in films 1 and 2 at  $I = 0$ . For weak Josephson coupling,  $\epsilon_J \ll \min(\epsilon_m/\xi_m^2)$ , the phase length  $L_\theta$  is much greater than  $\xi_1$  and  $\xi_2$ , except for special cases discussed below. From Eq. (3) we obtain the following dynamic equation for  $\theta(\mathbf{r}, t)$ :

$$\tau^2 \ddot{\theta} + \tau_r \dot{\theta} = L_\theta^2 \nabla^2 \theta - \sin \theta - \text{div}(\mathbf{Q} h) + \eta, \quad (7)$$

where  $\tau^2 = C\phi_0/2\pi c J_c$ ,  $\tau_r = \phi_0/2\pi c R J_c$ , and  $C$  and  $R$  are the sheet capacitance and quasiparticle ohmic resistance of the Josephson contact, respectively.

Eq. (3) resembles the free energy of a long Josephson contact in a magnetic field, but in our case the driving term  $(\mathbf{Q} \nabla \theta) h$  results from the pairbreaking asymmetry of the layers. Here  $h(Q) \propto I^2$  at small  $I$ , but  $h(Q)$  diverges at the global depairing current  $I_d = I(Q_d)$  where  $Q_d^2 = (\epsilon_1 + \epsilon_2)/3(\epsilon_1 \xi_1^2 + \epsilon_2 \xi_2^2)$ . Yet,  $I_d$  cannot be reached because the phase-locked state becomes unstable above the depairing threshold,  $Q > 1/\sqrt{3}\xi_2 < Q_d$ , in film 2.

At  $Q \approx 1/\sqrt{3}\xi_2$  the gradient term in Eq. (3) changes sign so  $G_\theta$  should be expanded in higher order spatial derivatives of  $\theta$ , which add the stabilizing term  $\ell^2 L_0^2 (\nabla^2 \theta)^2 / 2$  into Eq. (3) where  $L_\theta^2(Q) \approx L_0^2(1 - 3\xi_2^2 Q^2)$ ,  $L_0 = 2(\epsilon_2/\epsilon_J)^{1/2}$ , and  $\ell \sim \xi_2$ . In the critical region,  $Q \approx 1/\sqrt{3}\xi_2$ , a small perturbation  $\delta\theta = \theta_0 \cos kx$  changes the energy by  $\delta G_\theta \propto 1 - k^2 L_0^2(3\xi_2^2 Q^2 - 1) + \ell^2 L_0^2 k^4$ . Hence,  $\delta G_\theta$  is minimum at  $k_m^2 = (3\xi_2^2 Q^2 - 1)/2\ell^2$ , and  $\delta G_\theta(k_m) < 0$  if  $3\xi_2^2 Q^2 > 1 + 2\ell/L_0$ . The bilayer becomes unstable with respect to phase perturbations with the wave vector  $k_m = (\ell L_0)^{-1/2}$  at  $Q > Q_{c2}$  where

$$Q_{c2} \simeq \frac{1}{\sqrt{3}\xi_2} \left( 1 + \frac{\ell}{2} \sqrt{\frac{\epsilon_J}{\epsilon_2}} \right), \quad k_m \simeq \frac{\epsilon_J^{1/4}}{(2\ell)^{1/2} \epsilon_2^{1/4}}. \quad (8)$$

At the spinodal point  $Q = Q_{c2}$  the phase-locked state is absolutely unstable, but stable large-amplitude textures become energetically favorable at a lower  $Q \approx Q_{c1}$  due to proliferation of interlayer  $2\pi$  phase slips  $\theta = 4 \tan^{-1} \exp(x/L_\theta)$  similar to the Josephson vortices at  $H > H_{c1}$ . At  $Q = Q_{c1} \ll Q_{c2}$  the energy of a single phase slip  $8\epsilon_J L_\theta$  equals the gain in the condensation energy  $2\pi\epsilon_J Q_{c1} h(Q_{c1})$  in Eq. (3). Hence,

$$Q_{c1} = \left[ \frac{(\epsilon_1 + \epsilon_2)\epsilon_J}{\pi^2 \epsilon_1 \epsilon_2 (\xi_2^2 - \xi_1^2)^2} \right]^{1/6}, \quad (9)$$

For  $I > I_{c1} = 8\pi c Q_{c1} (\epsilon_1 + \epsilon_2)/\phi_0$ , the minimum energy corresponds to the chain of phase slips spaced by  $a(I)$  for which  $\sin \theta/2 = \text{cn}(x/p L_\theta | p^2)$  [9]. Here  $\text{cn}(x|p^2)$  is an elliptic function, and

$$a = 2L_\theta p K(p^2), \quad I^3 = I_{c1}^3 E(p^2)/p, \quad (10)$$

where  $K(p^2)$  and  $E(p^2)$  are the complete elliptic integrals defined by the parameter  $0 < p < 1$ . The period  $a(I)$  diverges logarithmically at  $I \rightarrow I_{c1}$ , but then  $a \approx 0.5\pi^2 L_\theta (I_{c1}/I)^3$  decreases rapidly as  $I$  further increases. As a result, the current density in layer 2 remains close to  $J_{2d}$ , while  $J_1$  increases up to  $J_{1d}$ , giving the maximum current  $I_d(T) = d_1 J_{d1}(T) + d_2 J_{d2}(T)$  for decoupled layers. Formation of the phase textures at  $Q = Q_{c1}$  is therefore a first order phase transition with a spinodal decomposition of the phase-locked state at  $Q > Q_{c2}$ .

The above GL theory of current-induced decoupling of spatially separated condensates holds if  $T_{c1}$  and  $T_{c2}$  are not too different, for example, in a bilayer made of the same superconductor with different concentrations of nonmagnetic impurities. It also implies that

$T < T_{c2} < T_{c1}$  so superconducting states in both layers are weakly coupled, unlike the case  $T > T_{c2}$  for which  $\Delta_2$  is induced by proximity effect, and the phase-locked state persists for all  $I < I_d$  [9]. Now we turn to the condensate decoupling in the momentum space, focusing on interband phase textures in two-gap superconductors. We use here the quasiclassic equations of two-gap superconductivity in the dirty limit [3], which also enable us to calculate the phase textures in bilayers for all  $T$ :

$$\omega f_m - \frac{D_m^{\alpha\beta}}{2} [g_m \Pi_\alpha \Pi_\beta f_m - f_m \nabla_\alpha \nabla_\beta g_m] = \Psi_m g_m + \Gamma_{m\bar{m}} (g_m f_{\bar{m}} - g_{\bar{m}} f_m), \quad (11)$$

$$\Psi_m = 2\pi T \sum_{\omega>0}^{\omega_D} \sum_{m'} \lambda_{mm'} f_{m'}(\mathbf{r}, \omega), \quad (12)$$

$$J^\alpha = -2\pi eT \text{Im} \sum_{\omega>0} \sum_m N_m D_m^{\alpha\beta} f_m^* \Pi_\beta f_m. \quad (13)$$

Here  $f_m(\mathbf{r}, \omega)$  and  $g_m(\mathbf{r}, \omega)$  are the Usadel functions in the  $m$ -th band,  $|f_m|^2 + g_m^2 = 1$ ,  $\omega = \pi T(2n+1)$ ,  $D_m^{\alpha\beta}$  are the intraband diffusivities,  $\bar{m} = 2$  if  $m = 1$ , and  $\bar{m} = 1$  if  $m = 2$ ,  $\Gamma_{m\bar{m}}$  are the interband scattering rates,  $\Pi = \nabla + 2\pi i \mathbf{A}/\phi_0$ ,  $N_m$  is the partial density of states,  $\lambda_{mm'}$  are the BCS coupling constants, and  $N_1 \lambda_{12} = N_2 \lambda_{21}$ . The indices 1 and 2 correspond to the  $\sigma$  and  $\pi$  bands of MgB<sub>2</sub> for which  $\lambda_{\sigma\sigma} \approx 3\lambda_{\pi\pi} \approx 8\lambda_{\sigma\pi}$ ,  $N_\pi \approx 1.3N_\sigma$  [15].

Eqs. (11)-(13) can be obtained by varying the free energy  $\mathfrak{F} = \int F d^3\mathbf{r}$  where

$$F = \frac{1}{2} \sum_{mm'} \Psi_m \Psi_{m'}^* N_m \lambda_{mm'}^{-1} + F_1 + F_2 + F_i + \frac{B^2}{8\pi}, \quad (14)$$

$$F_m = 2\pi T \sum_{\omega>0} N_m [\omega(1 - g_m) - \text{Re}(f_m^* \Psi_m) + D_m^{\alpha\beta} (\Pi_\alpha f_m \Pi_\beta f_m^* + \nabla_\alpha g_m \nabla_\beta g_m)/4]. \quad (15)$$

Here  $F_m$  is the intraband energy, and  $F_i = 2\pi T \text{Re} \sum_{\omega>0} (N_1 \Gamma_{12} + N_2 \Gamma_{21}) (g_1 g_2 + f_1^* f_2 - 1)$  is due to interband scattering. The first term in Eq. (14) contains the Josephson-like interband coupling energy  $-\epsilon_i \cos \theta$  where  $\epsilon_i = \Delta_1 \Delta_2 \lambda_{12} N_1 / w$ , and  $w = \lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21}$ .

We derive the equation for the slowly varying ( $L_\theta \gg \xi$ ) interband phase difference  $\theta(\mathbf{r})$ , neglecting weak interband scattering [10] and expanding Eqs. (11)-(15) in powers of  $Q^2$  and  $Q^4$ , where the term  $\propto Q^4$  accounts for current pairbreaking in the lowest order in  $J$ . This calculation gives the energy of the phase texture in the form of Eq. (3) with  $\eta = 0$  and

$$L_\theta^2 = \frac{2\varphi_1 \varphi_2}{(\varphi_1 + \varphi_2) \epsilon_i}, \quad h = 2Q^2 \frac{(\psi_2 \varphi_1 - \psi_1 \varphi_2)}{(\varphi_1 + \varphi_2) \epsilon_i}, \quad (16)$$

where  $Q = 8\pi^2 \Lambda^2 J/c\phi_0$ ,  $\Lambda = \phi_0/[32\pi^3(\varphi_1 + \varphi_2)]^{1/2}$  is

the London penetration depth [3],

$$\frac{\varphi_m}{N_m D_m} = \frac{\pi \Delta_m}{8} \tanh \frac{\Delta_m}{2T}, \quad (17)$$

$$\frac{\psi_m}{N_m D_m^2} = u_m + \frac{\pi^2}{32v_m} \left[ \tanh \frac{\Delta_m}{2T} + \frac{\Delta_m}{2T} \operatorname{sech}^2 \frac{\Delta_m}{2T} \right]^2. \quad (18)$$

Here  $u_m = \pi T \Delta_m^2 \sum_{\omega>0} \omega^2 / (\omega^2 + \Delta_m^2)^{5/2}$ ,  $v_m = 2\pi T \Delta_m^2 \sum_{\omega>0} (\omega^2 + \Delta_m^2)^{-3/2}$ . Eqs. (16)-(18) can also be applied to bilayers, by replacing  $J \rightarrow I$ ,  $\epsilon_i \rightarrow \phi_0 J_c / 2\pi c$ , and  $N_m \rightarrow d_m N_m$ . Eq. (16) reduces to Eqs. (4)-(6) near  $T_c$  if  $Q\xi_2 \ll 1$ . However, Eq. (16) also describes the case (particularly relevant to MgB<sub>2</sub>) for which  $\Psi_1$  and  $\Psi_2$  become weakly coupled only at low  $T$  as intraband pairing causes the Cooper instability in band 2. The simpler GL theory gives the full dependencies of  $L_\theta$  and  $h$  on current, while in the Usadel approach only the main quadratic term in  $h(Q)$  can be obtained analytically for all  $T$ . In particular, for  $T = 0$ , Eq. (16) yields

$$L_\theta^2 = \frac{\pi N_1 N_2 \Delta_1^2 \Delta_2^2 \xi_1^2 \xi_2^2}{4(N_1 \Delta_1^2 \xi_2^2 + N_2 \Delta_2^2 \xi_1^2) \epsilon_i}, \quad (19)$$

$$J_{c1} = \frac{k c \phi_0}{8\pi^2 \Lambda^2 [(\xi_2^2 - \xi_1^2) L_\theta]^{1/3}} \quad (20)$$

Here the band decoupling current density  $J_{c1}$  is defined as before by  $8L_\theta = 2\pi Q_{c1} h(Q_{c1})$  where  $h = (\pi/4 + 4/3\pi) L_\theta^2 (\xi_2^2 - \xi_1^2) Q^3$ ,  $Q_{c1} = 8\pi^2 \Lambda^2 J_{c1} / c\phi_0$ ,  $\xi_m = (D_m/\Delta_m)^{1/2}$  is the intraband coherence length, and  $k = (1/3 + \pi^2/16)^{-1/3} \approx 1.017$ . If expressed in terms of the sheet condensation energy densities  $\epsilon_m = N_m \Delta_m^2 \xi_m^2 / 2$ , Eqs. (19) and (20) reduce Eqs. (5) and (9) to the accuracy of coefficients  $\sim 1$ .

The results presented above indicate that the previous calculations of  $J_d$  [11, 12] for two-gap superconductors in which both bands were assumed to be phase-locked are only valid at higher temperatures  $T > T_2 \sim T_c \exp(1/\lambda - 1/\lambda_{22})$  for which  $\Delta_2$  is induced by interband coupling. Here  $\lambda = [\lambda_+ + (\lambda_-^2 + 4\lambda_{12}\lambda_{21})^{1/2}] / 2w$  and  $\lambda_\pm = \lambda_{11} \pm \lambda_{22}$ . To estimate  $T_2$  for MgB<sub>2</sub>, we take  $T_c = 40\text{K}$ ,  $\lambda_{11} = 0.8$ ,  $\lambda_{22} = 0.3$ ,  $\lambda_{12} = 0.12$ ,  $\lambda_{21} = 0.09$  [15] for which  $T_2 \simeq 0.12T_c \simeq 5\text{K}$ . This qualitative interpretation is consistent with the Usadel calculations [11] and the strong coupling Eliashberg theory [12], which predict a two-hump  $J(Q)$  at low  $T$ , which turns into the conventional dome-like  $J(Q)$  at higher  $T$ . For  $T > T_2$ , current pairbreaking thus occurs in the phase-locked state, for which  $J_d = \max(J(Q))$ .

A two-hump  $J(Q)$  at low  $T$  means that four possible phase gradients can provide the same current density  $J = J(Q)$ . Since only states with  $d|J|/dQ > 0$  are stable, the two-hump  $J(Q)$  would indicate formation of a stratified flow comprised of parallel channels with two different phase gradients, similar to the Gunn instability in semiconductors. However, this stratification is preceded by interband decoupling, since for  $\lambda_{12} \ll \lambda_{22}$ , the band 2 cannot sustain the same  $Q$  as the band 1. Thus, inter-

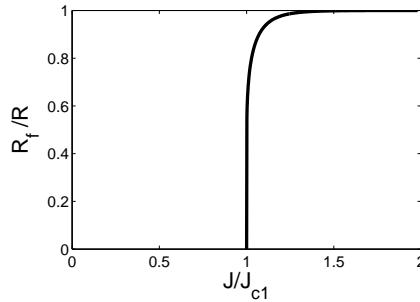


FIG. 2: Flux flow resistance  $R_f(I)$  for the moving phase texture as a function of the current  $I$  along the bilayer.

band current redistribution provided by the phase textures *increases* the GL depairing current density  $J_d(T)$  for which  $Q_1 = 1/\sqrt{3}\xi_1$  and  $Q_2 = 1/\sqrt{3}\xi_2$  as compared to its phase-locked value  $\tilde{J}_d = \max J(Q)$ :

$$\tilde{J}_d = \frac{8\pi c(\alpha_1\gamma_1/\beta_1 + \alpha_2\gamma_2/\beta_2)^{3/2}}{3\sqrt{3}\phi_0(\gamma_1^2/\beta_1 + \gamma_2^2/\beta_2)^{1/2}}, \quad (21)$$

$$J_d = \frac{8\pi c}{3\sqrt{3}\phi_0} \left( \frac{\alpha_1\sqrt{\alpha_1\gamma_1}}{\beta_1} + \frac{\alpha_2\sqrt{\alpha_2\gamma_2}}{\beta_2} \right), \quad (22)$$

where  $\alpha_m$ ,  $\beta_m$  and  $\gamma_m$  are the 2-gap GL expansion coefficients. The enhancement of  $J_d$  is most pronounced in the case of a clean  $\sigma$  band (large  $\gamma_1$  [3]) at low  $T < T_2$  where  $J_d(T)$  could be measured by a pulse technique [13].

Phase textures in bilayers manifest themselves in dc transport if both parallel and perpendicular currents  $I$  and  $I_\perp$  are applied, as shown in Fig. 1. For  $I < I_{c1}$ , no voltage across the bilayer occurs, but for  $I > I_{c1}$ , the oscillating voltage  $V(x, t) = -(\phi_0 v / \pi c p L_\theta) \text{dn}[(x - vt)/pL_\theta | p^2]$  and the ohmic average voltage  $\bar{V} = R_f I_\perp$  appear due to the phase slip structure moving with the velocity  $v(I, I_\perp)$ . Here the resistance  $R_f$  is similar to the flux flow resistance of a long Josephson contact [14]

$$R_f = \pi^2 R / 4K(p^2)E(p^2), \quad (23)$$

where  $R$  is the resistance of the interlayer contact, and  $p(I)$  is defined by Eqs. (10). The dependence  $R_f$  on the longitudinal current  $I$  shown in Fig. 2 describes switching between superconducting and resistive states across the bilayer. Other effects include a parametric resonance caused by superimposed ac currents  $I_\perp(t)$  and  $I(t)$ , since  $I(t)$  modulates the parameters in Eq. (7). Therefore, the geometry in Fig. 1 can provide switching and Josephson flux flow oscillator in current-operated devices.

Phase textures in two-gap superconductors can move due to *interband* currents produced, for example, by nonequilibrium charge imbalance [7]. One could also expect a kink in the nonlinear low-frequency ( $\omega \ll \Delta_2$ ) rf surface resistance as the field amplitude  $H_0$  exceeds the onset of the interband phase slip formation  $H_\theta =$

$4\pi J_{c1}\Lambda/c$ . To estimate  $H_\theta$  for MgB<sub>2</sub>, we assume that the interband breakdown occurs as the screening current density  $J(0) \simeq cH_0/4\pi\Lambda$  on the surface produces the phase gradient  $Q = 8\pi^2\Lambda^2 J(0)/c\phi_0$  exceeding the GL depairing limit  $Q_{c2} = 1/\sqrt{3}\xi_\pi$  in  $\pi$  band. Hence,  $H_\theta = \phi_0/2\sqrt{3}\Lambda\xi_\pi \sim H_c\xi_\sigma/\xi_\pi$ , where  $H_c$  is the thermodynamic critical field, and the ratio  $\xi_\sigma/\xi_\pi$  can be strongly affected by impurities [3]. Taking  $\xi_\sigma/\xi_\pi \approx 0.3$  for MgB<sub>2</sub> single crystals [16] and  $H_c(0) \simeq 0.3$ T, we find  $H_\theta(0) \sim 0.1$ T. Such fields cause breakdown of the linear London electrodynamics, affect properties of vortex lattice, penetration vortices through surface barrier, etc.

In conclusion, two weakly coupled co-moving superconducting condensates can undergo a first order phase transition into a phase textured state well below the global depairing current. Such textures controlled by current result in resistive switching and oscillating effects.

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*Note added.* After this paper was submitted, a phase textured state with two different winding numbers in weakly coupled Al rings has been observed [17].

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